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EVAPORATION OF A LIQUID DROP IMMERSED IN ANOTHER IMMISCIBLE LIQUID. THE CASE OF $\sigma_c < \sigma_d$

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Abstract—This paper presents the analytical description of the mechanism of the evaporation of the liquid drop immersed in another immiscible liquid medium. The above mechanism is related to the case when the dispersed phase surface tension is greater than the continuous phase surface tension. Thus, as a result of the drop overheating, the vapour nuclei move outside of the drop. The drop size is reduced. This process continues to the moment, when the drop is so overheated, that a thermal explosion takes place. The results of the mathematical description are compared with experimental data.

NOMENCLATURE

a_1, a_2,	constants in equations (16) and (17);
b_1, b_2, b_3,	constants of equations (14) and (15);
C,	constant of equation (1);
E,	reduced temperature;
k,	Boltzmann constant;
L,	heat of evaporation [cal/g];
m,	coefficient [dimensionless];
n,	number of molecules in nucleus;
p,	pressure [G/cm^2];
r,	polar co-ordinate;
R_0,	drop radius [cm];
R_p,	nucleus radius [cm];
$\dot{R}_p$,	dR_p/dt [cm/s];
T_c,	continuous phase temperature [K];
T_s,	saturation temperature [K];
T,	temperature [K];
U,	drop velocity in relation to continuous phase [cm/s];
V_r,	radius component of velocity [cm/s];
V_θ,	angle component [cm/s];
V'',	specific vapour volume [cm^3/g];
V,	Cartesian co-ordinate (reduced);
Z,	Cartesian co-ordinate (reduced);
X,	Cartesian co-ordinate;
Y,	Cartesian co-ordinate.

Greek symbols

α,	thermal diffusivity [cm^2/s];
λ,	thermal conductivity [cal/m °C s];
τ,	time;
σ,	surface tension [dyn/cm];
σ_c,	surface tension of the continuous phase;
σ_d,	surface tension of dispersion phase;
θ,	angle co-ordinate;
μ,	viscosity [g/cm s].

INTRODUCTION

IN OUR previous work introducing the problems of the mechanism of evaporation of a liquid drop immersed in the overheated (with respect to its boiling point) immiscible liquid medium we have shown [1] that depending on the ratio of the surface tensions of both liquids there must be at least two completely different mechanisms of evaporation.

The case of $\sigma_c > \sigma_d$ connected with the evaporation of liquid into a drop which thus takes the shape of a flat sheath suspended on the expanding vapour bubble is described in our previous paper [2]. The second case of $\sigma_c < \sigma_d$ connected with the removal of the creating vapour nuclei from the overheated surface layer of a drop to the surrounding liquid is the subject of this paper.

DESCRIPTION OF MECHANISM

Let us consider a continuous phase uniformly heated to the temperature T_c into which we introduce the liquid drop, which is heated to the temperature of its boiling point T_s and $T_c > T_s$.

The sloping or rising liquid drop receive a certain level of overheating from the surrounding continuous phase and thus pass to the metastable equilibrium state.

In the metastable equilibrium state the regions of the created new phase (by means of the phase fluctuations) must have a minimal size, being the condition which is greater than the initial stability of the newly-created phase. The smaller size secondary phase is created as the result of the fluctuations disappearing. The new phase region of minimal size becomes the vapour nucleus in the equilibrium state with the initial phase. Using the fluctuation theory conceptions and assuming the spherical nucleus we can derive its creation probability.

On the base of the formula from [3]

$$W = C \cdot \exp\left\{-\frac{16\sigma^3 T_s^2 V''^2}{3kT(T-T_s)^2 L^2}\right\} \quad (1)$$

where:

C = constant
σ = liquid–vapour surface tension
V'' = specific volume of vapour phase
k = Boltzmann constant

L = heat of phase transformation

T_s = boiling point

T = temperature of the overheated liquid phase.

As can be seen from [1] the creation of the new phase nucleus in the pure liquids depends on their overheating.

In order to consider the circumstances connected with the overheating of the drop during its motion in the continuous phase we shall analyse the equation of heat transport to the drop.

The spherical drop is moving with the relative rate U (Fig. 1) in the counter-current with respect to the continuous phase.

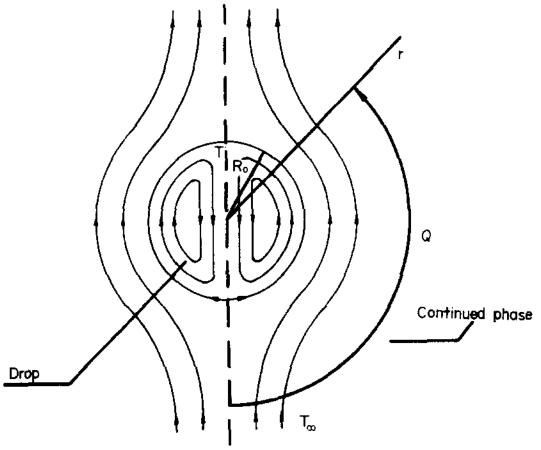

FIG. 1. Water drop falling down in silicon oil. The sketch for the analytical description.

Taking into consideration the spherical drop symmetry during the motion of the drop, the equation of the heat transport in the polar co-ordinates can be derived in the form:

$$\frac{\partial T}{\partial \tau}+V_r\frac{\partial T}{\partial r}+V_\theta\frac{1}{r}\frac{\partial T}{\partial \theta}=\alpha\Delta T \qquad (2)$$

where:

T = temperature

τ = time

V_r = radius component of the velocity vector

V = angle component

α = thermal diffusivity coefficient

Δ = Laplacian operator

r = radius coordinate

θ = angle coordinate.

Equation (2) in its general form is valid for the continuous phase, too. In order to express the detailed equations for both phases, the analytical form of the components V_r and V for the drop and the external continuous phase ought to be found. For this purpose we shall consider the hydrodynamics of the drop motion.

The drop is moving in the continuous phase influenced by the gravity force. Its motion can be taken as potential.

The motion of the liquid inside and outside of the drop is described by the identity equations of Navier and Stokes [3] and the continuity equation (4).

For the continuous phase:

$$\operatorname{grad} p = \mu\Delta V \qquad (3)$$

$$\operatorname{div} V = 0. \qquad (4)$$

For the drop:

$$\operatorname{grad} p' = \mu'\Delta V' \qquad (5)$$

$$\operatorname{div} V' = 0 \qquad (6)$$

where:

p = pressure

V = velocity

μ = viscosity.

The prime relates to inside the drop.

Based on the definition of the differential operators and on the assumption of spherical symmetry the set of equations (3) and (6) may be described in the polar co-ordinates.

The limiting conditions in this system of co-ordinates are as follows:

$$V_r \to U\cos\theta \qquad (7)$$

$$V_\theta \to -U\sin\theta. \qquad (8)$$

On the phase division surface the stress tensor components resulting from the phase boundary friction, like the normal component p_{rr} and the tangent component Pr_θ, remain constant.

In the spherical system it is represented by the equations in the form:

$$-p+2\mu\frac{\partial V_r}{\partial r}+P=p'+2\mu'\frac{\partial V_r'}{\partial r} \qquad \text{for } r=R_0 \qquad (9)$$

$$\mu\left(\frac{1}{r}\frac{\partial V_r}{\partial \theta}+\frac{\partial V_\theta}{\partial r}-\frac{V_\theta}{r}\right)=\mu'\left(\frac{1}{r}\frac{\partial V_r'}{\partial r}+\frac{\partial V_\theta'}{\partial r}-\frac{V_\theta}{r}\right)$$

$$\text{for } r=R_0. \qquad (10)$$

Besides the known symbols P is the effective pressure provoking the motion.

On the drop surface other conditions ought to be realized

$$V_r'=0 \qquad r=R_0 \qquad (11)$$

$$V_r=0 \qquad r=R_0 \qquad (12)$$

$$V_\theta=V_\theta' \qquad r=R_0. \qquad (13)$$

From the set of the equations (3)–(6) with the conditions (7)–(13) we can univocally calculate the components V_r, V_θ, V_r' and V_θ'.

The above set of the equations let us also calculate the current function described by Hadamard [4] and Rybczyński [5].

We will be interested in the calculation of the components V_r, V_θ, V_r', V_θ' and of the velocity U of the moving drop. These values are necessary for the description of the heat-transfer equations.

The above mentioned limiting conditions (7) and (8) show that the solutions of the set of the equations (3)–(6) will be in the form:

$$V_r=f(r)\cos\theta, \quad V_\theta=g(r)\sin\theta.$$

Similarly for the drop inside, the velocities V_r and V_θ as the functions of the variables r and θ can be expressed in the form of the functions product, in which every one of the functions is dependent only on one parameter. The set of partial differential equations is resolved into the set of the normal equations and then into the set of algebraic equations.

Such a procedure is presented in [6].

The analysis of the set of the equations and the limiting conditions gives the velocity components in the form:

$$V_r = \left(\frac{b_1}{r^3}+\frac{b_2}{r}+b_3\right)\cos\theta \tag{14}$$

$$V_\theta = \left(-\frac{b_1}{2r^3}+\frac{b_2}{2r}-b_3\right)\sin\theta \tag{15}$$

$$V_r = (a_1 r^2 + a_2)\cos\theta \tag{16}$$

$$V_\theta = -(2a_1 r^2 + a_2)\sin\theta \tag{17}$$

where a_1, a_2, b_1, b_2, b_3 are constants.

Using the forms (14)–(17) and the conditions (7)–(13) we can find a_1, a_2, b_1, b_2 and b_3.

After the calculation of the components V_r, V_r', V_θ, V_θ' and U the equation (2) takes the following form for the external region of the drop:

$$\begin{aligned}\frac{\partial T}{\partial\tau} &= U\cos\theta\left[1-\frac{R_0}{2r}(a-1)-\frac{R_0^3}{2r^3}(3-a)\right]\frac{\partial T}{\partial r}\\ &\quad + U\sin\theta\left[1+\frac{R_0}{4r}(a-1)-\frac{R_0}{4r^3}(3-a)\right]\\ &= \alpha\Delta T\end{aligned} \tag{18}$$

where:

$$U = \frac{2}{3\mu} g R_0^2 \frac{\mu+\mu'}{3\mu+2\mu'}(\rho'-\rho) \tag{19}$$

$$a = \frac{\mu}{\mu+\mu'}. \tag{20}$$

The conditions of the equation (18) are:

$$\tau = 0, \quad T = T_\infty, \quad T(r,\theta,0) = T \tag{21}$$

$$0 \leqslant \theta \leqslant 2\pi, \quad r = R_0, \quad T(R_0,\theta,\tau) = T_i \tag{22}$$

$$0 \leqslant \theta \leqslant 2\pi, \quad r = \infty, \quad T(\infty,\theta,\tau) = T_\infty \tag{23}$$

where T is the temperature of continuous phase a great distance away from the drop ($r > 4R_0$) and T_i is the temperature on the phase boundary.

The solutions (18)–(23) give the temperature distribution round the drop. There is a relationship between the external region of the drop discussed here and, more important for us from the point of view of mechanism, the problem of the internal part of the drop. They are connected by the condition on the phase boundary.

We shall write now the internal problem with its conditions. For convenience of the numerical calculations we shall define the new values and then we shall introduce them to equation (2).

$$E = \frac{T-T_s}{T_i-T_s} \tag{24}$$

$$x = \frac{r}{R_0} \tag{25}$$

where T_s is the boiling point of liquid forming the drop and T is the temperature inside the drop.

The equation (2) after taking into consideration the equations (23) and (24) and (16) and (17) has the form:

$$\begin{aligned}\frac{\partial E}{\partial\tau} &= \left(\frac{2\alpha}{xR_0^2}-\frac{U(1-x^2)\cos\theta}{2R_0}\right)\frac{\partial E}{\partial x}\\ &\quad+\left[\frac{\alpha\cos\theta}{x^2R_0^2\sin\theta}+\frac{U(1-x^2)}{2R_0x}\right]\frac{\partial E}{\partial\theta}\\ &\quad+\frac{\alpha}{R_0^2}\cdot\frac{\partial^2E}{\partial x^2}+\frac{\alpha}{x^2R_0^2}\cdot\frac{\partial^2E}{\partial\theta^2}\end{aligned} \tag{26}$$

$$0 \leqslant x \leqslant 1$$
$$0 \leqslant \theta \leqslant \pi.$$

The initial and boundary conditions have the form:

$$E(x,\theta,0) = 0 \tag{27}$$

$$E(1,\theta,\tau) = 1 \tag{28}$$

$$\frac{\partial E}{\partial x}(0,\theta,\tau) = 0 \tag{29}$$

$$E(x,0,\tau) = E(x,\pi,\tau) \tag{30}$$

$$\frac{\partial E}{\partial\theta}(x,0,\tau) = \frac{\partial E}{\partial\theta}(x,\pi,\tau). \tag{31}$$

For the convenience of numerical solution the transformation of the problem into the reduced Cartesian coordinates was made when

$$Z = \frac{X}{R_0}, \quad V = \frac{Y}{R_0}$$

we have:

$$\begin{aligned}\frac{\partial E}{\partial\tau} &= \frac{U}{2R_0}(Z^2+2V^2-1)\frac{\partial E}{\partial Z}-\frac{U}{2R_0}ZV\frac{\partial E}{\partial V}\\ &\quad+\frac{\alpha}{R_0^2}\left(\frac{\partial^2E}{\partial Z^2}+\frac{\partial^2E}{\partial V^2}\right)\end{aligned} \tag{32}$$

$$E(Z,V,0) = 0 \tag{33}$$

$$E(Z,V,\tau) = 1, \quad Z^2+V^2 = 1 \tag{34}$$

$$\left.\frac{\partial E}{\partial V}\right|_{V=0} = 0. \tag{35}$$

The problem (32)–(35) was resolved by numerical methods on the computer IBM 370. In the solution, which will be discussed below, a five point open diagram was used. The stability of the problem's solution for the class of functions which are coefficients of partial derivatives in equation (32) was proved in [7] (see also the Appendix).

DESCRIPTION OF MECHANISM OF NEW INITIAL CONDITIONS FORMULATION

As was mentioned in the formulation of the conditions of the equation (32), initially the drop is heated to the temperature $T = T_s (E = 0)$.

During all the time of the drop motion the temperature on the surface $T = T_i (E = 1)$. During the motion the drop is warming; the drop temperature distribution—for fixed $\theta = \pi/4$—along the radius, for the

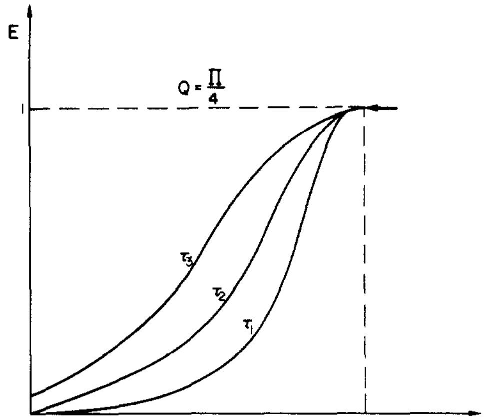

FIG. 2. The temperature distribution within a drop along the radius for different time ($\tau_1 < \tau_2 < \tau_3$) at constant angle $\theta = \pi/4$.

different periods of time is represented in Fig. 2. The presented curves are the results of the solution of the equations (32)–(35) up to critical nucleus creation. The size of such nucleus may be derived from the equation obtained based on the considerations of the statistical physics and the Clausius–Clapeyron equation (2)

$$R^* = \frac{2\sigma T_s V''}{L(T-T_s)}$$

where besides the known symbols, L is the heat of evaporation and R^* is the radius of nucleus.

In the heating drop zone, when the new phase is not yet created, the greater probability of nucleus creation in accordance with the formula (1) and the equations (32)–(35), is on the surface of phase division. The structure of the interfacial surface by its heterogeneity facilitates additionally the nucleus creation [8]. We are interested in the nuclei whose sizes are near to the critical, regarding their stability.

We are not concerned with the displacement of the middle of the nucleus during its growth. It is assumed that the middle of the nucleus is on the distance R^* from the boundary of the drop. This reduction (simplification), which leads into a small mistake because of a small value of R^*/R, enables us to solve this problem numerically. The middle of the nucleus is located at the lattice nearest to the boundary of the drop node built for the differential scheme.

The nucleus is created at the moment when the relative temperature is

$$E = m \cdot E_{\max}$$

where m is constant and $E_{\max}$ is the temperature on the drop boundary on the side of the continuous phase.

For the different values of m the creation probability of the nucleus is different. Assuming the value of m for fixed continuous phase temperature, we can compare the obtained result with the experiment.

After the nucleus creation on the surface delimited by the isotherm E, the nuclei are coalesced and at once leave the drop in the form of bubbles. The drop size decreases.

In the region of the created nucleus there is a decrease of temperature resulting from the heat transfer from the surroundings. At the moment that the nuclei leaves the drop, the new temperature profile becomes stabilized. The newly created temperature distribution is a new initial condition for the subsequent act of the nucleus creation through the repeated drop heating to the "critical" state.

For the description of the temperature distribution in the region of the creation and growth of the nuclei we shall consider the equation of the heat penetration to the growing nucleus.

This problem was considered, among others, in the papers [9] and [10]. The authors have resolved the energy balance equation for all regions of the growth of the nucleus. In this paper we try to find the temperature distribution for the nucleus growth in the region of its critical size.

The nucleus growth is the result of the heat penetration from the region near the nucleus boundary to the nucleus.

The nucleus growth equation for the incompressible liquid has the form:

$$\rho R_p \ddot{R}_p + \tfrac{3}{2}\rho \dot{R}_p^2 = \Delta p - \frac{2\sigma}{R_p} \tag{36}$$

where R_p is nucleus radius, $\dot{R} = \mathrm{d}R/\mathrm{d}t$; ρ is liquid density, $\ddot{R} = \mathrm{d}^2R/\mathrm{d}t^2$; Δp is overpressure in the nucleus in relation to the surrounding liquid and σ is surface tension.

Coupling the nucleus growth equation with the heat transfer equation we shall get:

$$\frac{\partial T}{\partial \tau} + \frac{R_p^2 \cdot \dot{R}_p}{r^2}\frac{\partial T}{\partial r} = \alpha \Delta T. \tag{37}$$

With the following conditions:

$$T(r, 0) = T_v \tag{38}$$

$$T(\infty, t) = T_v \tag{39}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R_p} = \frac{\dot{n}L}{4\pi\lambda R_p^2} = \frac{\rho'' L}{\lambda}\dot{R}_p \tag{40}$$

where T_v is temperature round the creating nucleus; τ is time; ρ'' is vapour density; α is thermal diffusivity coefficient; λ is thermal conductivity of liquid; $\dot{n} = \mathrm{d}n/\mathrm{d}t$; and n is the number of molecules in the bubble with the radius R_p.

To begin with, the coordinate system is placed in the middle of the nucleus. The characteristic time of the heat transmission in the problem discussed is the range of R_p^2/α. From the other side, the nucleus growth time is characterized by the value $R_p/\dot{R}_p$. We are interested in the range of the critical size of the nucleus radius ($R_p \approx R^*$) when its growth is smaller (the R_p value is small [9]).

As a result we have the inequality:

$$\frac{R_p^2}{\alpha} \ll \frac{R_p}{\dot{R}_p}. \tag{41}$$

The inequality enables us to omit the derivative $\partial T/\partial \tau$ in the equation (37). In the critical size region of the nucleus the expression

$$\frac{R_p^2 \dot{R}_p}{r^2}\cdot\frac{\partial T}{\partial r}$$

may also be omitted; if we consider (40), the above expression is of the second order, infinitesimal for the very small values of R_p. The equation (37) simplifies to the form:

$$\Delta T = 0. \tag{42}$$

By solving the equation (42) with the conditions (39) and (40) we get the equation in the form:

$$T = T_v - \frac{\rho'' L}{\lambda} \dot{R}_p R_p \text{ for } R_p \approx R^*. \tag{43}$$

Therefore in the region covered by the values of nucleus growth up to critical, there is a decrease of temperature according to the equation (43).

1. It is assumed that the heat used for the nucleus growth is taken from the drop (phase transformation). On the phase boundary, on the side of the continuous phase, the temperature remains constant.

After nucleus creation the temperature inside the drop remains the same as it is during the forming of the nucleus. It is caused by the smaller temperature conductivity coefficient for steam than for liquid.

2. In the region covered by the nucleus growth the temperature is decreasing (43).

3. In the remaining part of the drop in the middle direction the temperature profile is the same as in the moment of the beginning of the nucleus creation.

According to the argumentation discussed above after each act of the creation and the leaving off the drop by the nucleus the new initial situation for the numerical problem is created. The exemplifying temperature distribution at this moment is presented in Fig. 3. After each act of evaporation the drop size decreases, the drop is heating once more to the state ensuring the repeated nucleus creation.

At a certain moment the drop is overheated to such a level that the quantity of heat stored in it is equal to the quantity of heat necessary for the urgent evaporation (explosion) of the whole mass of liquid building the drop. The mechanisms of evaporation at the explosion moment are not the subject of this paper.

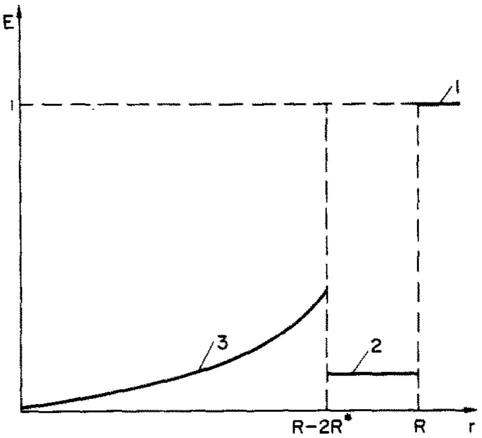

FIG. 3. The temperature distribution within a drop at the moment of leaving off the drop by the nuclei.

EXPERIMENTAL PART

The discussed mechanism was verified experimentally.

The measurements were carried out for the liquid system: (1) silicon oil—SILOL 50—as the continuous phase and (2) water—as the dispersed phase.

The measuring equipment in which the experiments were done is presented in Fig. 4.

The essential element of the equipment is the glass cylinder 1 with the thermal sheath. There is the silicon oil on the height H in the cylinder, which is heated to the fixed temperature T in the thermostats 7. The oil level is stabilized by means of the overflow 2.

Into the continuous phase prepared in this way we introduce the water drop by means of the calibrated

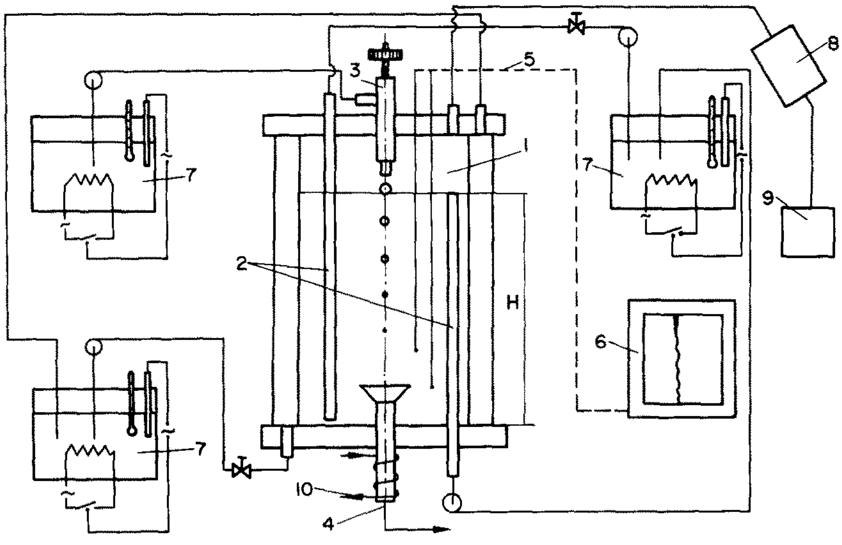

FIG. 4. The scheme of the measuring equipment.

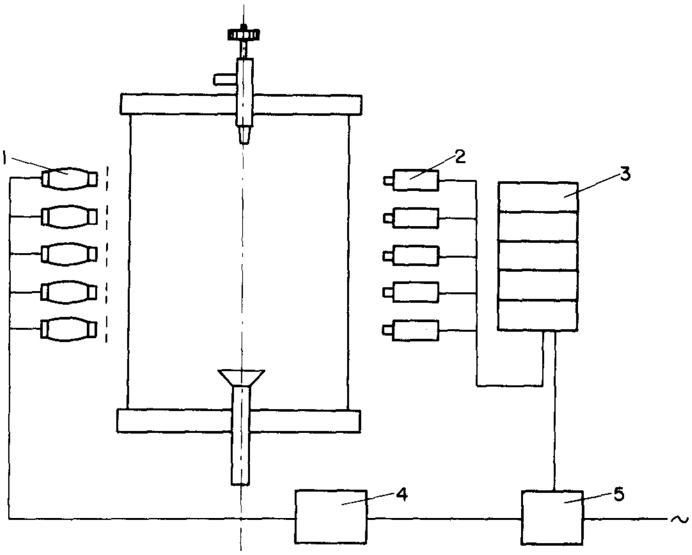

FIG. 5. The scheme of the optical system.

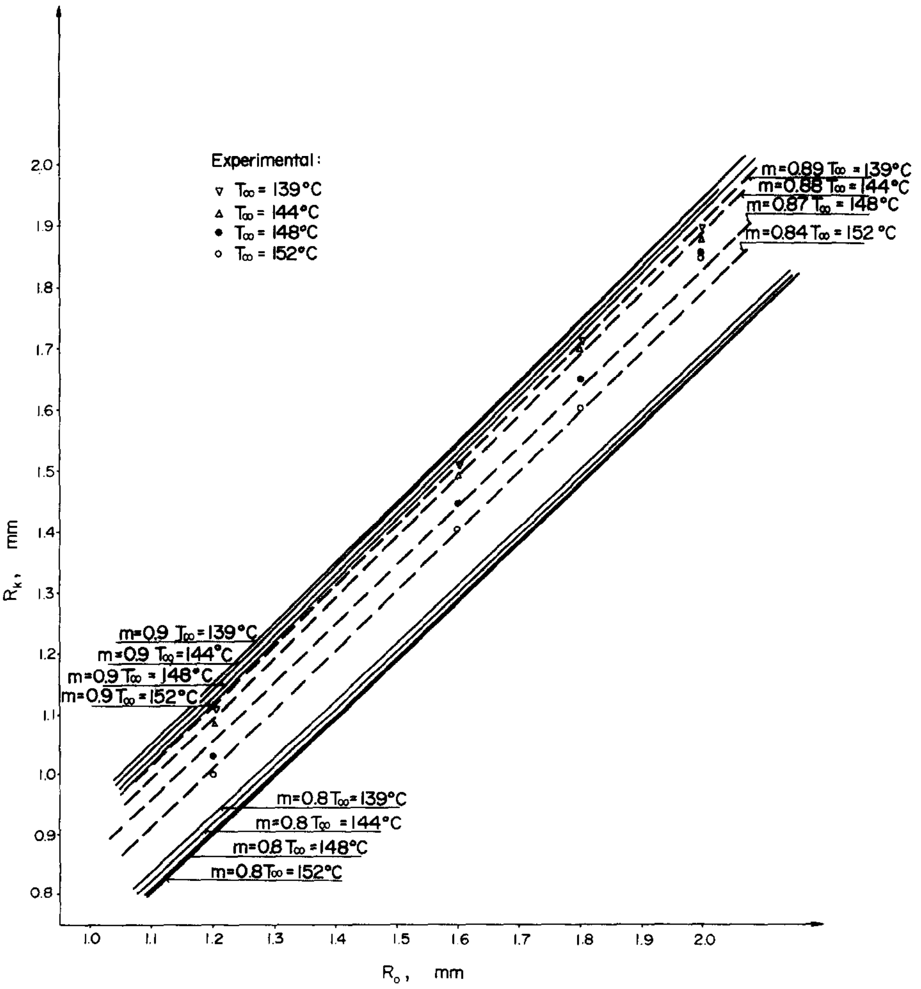

FIG. 6.

dropper 3, which was previously heated to its boiling point in the thermostat 7. After the partial evaporation the drop is collected in the receiver 4 and then weighted. The temperature is measured by means of the thermocouple system 5.

The drop motion is observed in the period from the moment of entering the continuous phase to the moment of receding from view in the receiver. The crossing time is recorded by means of a photodiodes system linked with the time recorder. This equipment is presented in the Fig. 5.

The experiment was carried out for different drop sizes and different continuous phase temperatures.

DESCRIPTION OF THE RESULTS

The results of the experiments are shown in Fig. 6. The relationship, between the terminal radius of the drop R_k (the radius of the drop before the explosion) and the beginning radius of the drop R_0 for the different temperatures of the continuous phase, is given there. Broken lines are for experimental data.

Continuous lines, different for different temperatures T and m, are for theoretical models.

Lines running through the experimental points are led by interpolation between theoretical lines.

As can be seen from the drawing there are values m (different for different overheating) for which good convergence between theoretical model and experimental data exists.

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APPENDIX

In numerical solution of problems (32)–(35) the five point open diagram was used (Fig. 1)

$$\begin{gathered}(i, \dot{s}+1)\\(i-\dot{1}, s)(\dot{i}, s)(i+\dot{1}, s)\\(i, \dot{s}-1).\end{gathered}$$

The partial derivatives for the spatial variable z (indicator i) are approximated in the following manner:

$$\frac{\partial^2 E}{\partial x^2}=\frac{E_{i-1,j,s}-2E_{i,j,s}+E_{i+1,j,s}}{2}$$

$$\frac{\partial E}{\partial x}=\frac{E_{i+1,j,s}-E_{i-1,j,s}}{2h}.$$

Similarly the partial derivatives of the variable V (indicator j).

The time derivative is approximated as follows (indicator s):

$$\frac{\partial E}{\partial \tau}=\frac{E_{i,j,s+1}-E_{i,s}}{k}$$

where h is spatial step and k is time step.

We note:

$$\xi \equiv \frac{k}{h^2}.$$

If we have the equation in the form:

$$\frac{\partial E}{\partial \tau}=a_1(x, y)\frac{E}{x^2}+a_2(x, y)\frac{\partial 2E}{\partial y^2}+b_1(x, y)\frac{\partial E}{\partial x}+b_2(x, y)\frac{\partial E}{\partial y}$$

l_x and l_y will be given the following:

$$l_x \leqslant \frac{2\min a_1}{\max b_1}, \qquad l_y \leqslant \frac{2\min a_2}{\max b_2}$$

and

$$\alpha_x=\frac{l_x}{h}, \qquad \alpha_y=\frac{l_y}{h}.$$

The proposed method of the approximation is stable when

$$\xi=\frac{1}{2\left(\frac{a_1}{\alpha_x^2}+\frac{a_2}{\alpha_y^2}\right)}$$

and $h\min(l_x, l_y)$.

EVAPORATION D'UNE GOUTTE LIQUIDE PLONGEE DANS UN LIQUIDE DIFFERENT NON MISCIBLE. CAS $\sigma_c < \sigma_d$

Résumé—L'article présente une description analytique du mécanisme d'évaporation d'une goutte liquide plongée dans un milieu liquide différent non miscible. Le mécanisme ci-dessus se rapporte au cas où la tension superficielle de la phase dispersée est supérieure à la tension superficielle de la phase continue. Les noyaux de vapeur créés à la surface surchauffée de la goutte sont éjectés dans le fluide environnant. La dimension de la goutte est réduite. Ce processus se poursuit jusqu'au moment où la goutte devient tellement surchauffée que l'explosion thermique se produit. Les résultats de la description mathématique sont comparés aux données expérimentales.

DIE VERDAMPFUNG EINES FLÜSSIGKEITSTROPFENS IN EINER ANDEREN, NICHT MISCHBAREN FLÜSSIGKEIT FÜR DEN FALL $\sigma_c < \sigma_d$

Zusammenfassung—Der Mechanismus der Verdampfung eines Flüssigkeitstropfens in einer anderen, nicht mischbaren Flüssigkeit wird analytisch beschrieben. Es wird der Fall betrachtet, daß die Oberflächenspannung der dispersen Phase größer als diejenige der kontinuierlichen Phase ist. Infolge der durch die Überhitzung bedingten Dampfkeime nimmt der Tropfendurchmesser ab. Dieser Vorgang setzt sich solange fort, bis der Tropfen eine solche Überhitzung besitzt, daß eine thermische Explosion stattfindet. Die Ergebnisse der mathematischen Berechnung werden mit experimentellen Daten verglichen.

ИСПАРЕНИЕ КАПЛИ ЖИДКОСТИ, ПОГРУЖЕННОЙ В ДРУГУЮ НЕСМЕШИВАЮЩУЮСЯ ЖИДКОСТЬ, ДЛЯ СЛУЧАЯ $\sigma_c < \sigma_d$

Аннотация — Аналитически описывается механизм испарения капли жидкости, погруженной в другую несмешивающуюся с первой жидкую среду. Механизм испарения описывается для случая, когда поверхностное натяжения в дисперсной среде больше, чем в однородной фазе. Тогда зародыши пара, возникающие в результате перегрева капли, вырываются за пределы капли, и капля уменьшается в размерах. Этот процесс продолжается до тех пор, пока капля не перегревается настолько, что происходит тепловой взрыв. Аналитические результаты сравниваются с экспериментальными данными.